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(6) THE RESISTANCE OF PIPE BENDS,

(10) by H./Lorenz (11) Nov 55 (12) 4p.

The resistance to motion i.e. the pressure drop in fluids when passing through a pipe bend (figure) has been the subject of numerous investigations since the work of Weisbach. His results show such a wide variation that it is difficult to fit them to a curve or to represent them in an empirical formula. All that is definite is that the resistance, for a pipe of circular cross-section with a 90° bend, is a minium when the ratio of the radius of the pipe to the average radius of curvature is approximately 1/7 to 1/8.

I. If the flow is in a closed circular ring, that is with a 360° bend, a potential motion will set in -neglecting wall friction- according to the rule

$$u r = u_0 r_0 \qquad (1)$$

where u, u_0 are the circuital velocities and r, r_0 are the pertinent trajectory radii. Even flow in bends of smaller degree, [illegible], approximates this type of motion, however, without being able to attain it completely. The same is true also for the radial pressure increase resulting from the centrifugal accelleration, which for a closed circular ring without an average radial velocity is given by

$$\frac{\partial p}{\partial r} = \frac{\gamma}{g} \frac{u^2}{r} \qquad (2)$$

where γ is the bulk density and g the acceleration of gravity. Integration of this between r_1, r_2, using (1), gives the pressure increase from the inside to the outside.

$$p_2 - p_1 = \frac{\gamma}{g} u_0^2 r_0^2 \int_{r_1}^{r_2} \frac{dr}{r^3} = \frac{\gamma}{2g}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) u_0^2 r_0^2 \qquad (2a)$$

This can also be written

$$p_2 - p_1 = \frac{\gamma}{2g}(u_1^2 - u_2^2) = \frac{\gamma}{g}(r_2 - r_1) u_0^2 \frac{(r_1 + r_2)}{2 r_1^2 r_2^2} r_0^2 \qquad (3)$$

or with the approximation

$$2 r_1^2 r_2^2 \approx (r_1 + r_2) r_0^3; \quad p_2 - p_1 = \frac{\gamma}{g} \frac{u_0^2}{r_0} (r_2 - r_1) \qquad (2b)$$

Flow in the cross-section now results from this pressure drop, directed toward the inner side of the ring, because of the adhesion (to the wall) in the neighborhood of the wall. According to figure I, this results in a double vortex due to the back flow in the center. This fact has been established by various observers. The right side of (2b) showsthe vortex energy per unit mass flowing through the ring. This energy is eventually annuled by the inner resistance, which produces a pressure drop in the direction of flow of an amount obtained directly from (2b).

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In this case, u_o can be considered as the mean velocity, r_o as the radius of curvature of the bend, and $r_2 - r_1 = 2a$ as the largest width of the bend's cross-section taken in the plane of motion. If the bend consists of only one arc ϕ, there is not a complete formation of the double vortex in the entire arc ϕ. This is because of the transition from straight flow at the entrance and the transformation back again at the exit. Therefore, a pressure drop due to vortex formation can only be expected to be

$$\Delta p = \frac{\phi}{2\pi}(p_2 - p_1) = \frac{\gamma}{g}\,\frac{u_o^2 a}{r_o}\,\frac{\phi}{\pi}. \qquad (4)$$

II. The velocity distribution given by (1) does not naturally originate at the entrance cross-section and accordingly does not go into an identical straight flow at the exit. Therefore, the volume flow per unit time through the cross-section F, is

$$u_o F = \int u\, dF = u \cdot r \int \frac{dF}{r} \qquad (5)$$

or approximating with $r = r_o + x$

$$u_o F = u \cdot r \int \frac{dF}{r_o + x} \qquad (5a)$$

$$\approx \frac{u \cdot r}{r_o}\int\left(1 - \frac{x}{r_o} + \frac{x^2}{r_o^2}\right) dF$$

III. If r_o is the radius of curvature of the flow lines through the center of gravity of the cross-section, and k the radius of gyration, then

$$\int x\, dF = 0 \qquad \int x^2 dF = k^2 F$$

in order that the axis of the center of gravity be perpendicular to the plane of motion. Using this, we obtain from (5a) with $u = u_o$:

$$r_o = r\left(1 + \frac{k^2}{r_o^2}\right) \qquad (5b)$$

Thus a shifting of the center stream-line toward the innerside is

$$r_o - r = \frac{k^2 r}{r_o^2} \approx \frac{k^2}{r_o} \qquad (6)$$

from this, with angle of curvature ϕ, only the expression

$$\Delta r = (r_o - r)\frac{\phi}{2\pi} = \frac{\phi}{2\pi}\,\frac{k^2}{r_o} \qquad (6a)$$

remains. This produces a narrowing of the flow toward the inner side of the bend, a separation from the outerside, near the entrance. Opposite circumstances occur at the exit. Thus the transition of both forms of flow into one another, as in the figure, may be found approximately at the center of the bend.(arc). Each flow separation is combined with a reverse flow at the boundary inside a vortex layer. This is clearly noticeable (figure) on the outside at the entrance and on the inside extending from the center to the exit of the bend. This has been repeatedly established in photographs by observers. This outer separation (up to the center of the bend and inner separation (from this point on) of the flow means that the pressure drop (4) is most pronounced from the outside toward the inside at the center. The pressure drop strongly diminishes after the bend and vanishes in the straight flow.

Either of the
~~The two~~, one-sided, half bend, flow separations can be considered as joint parts (over the half bend) of the total flow, Either separation having a separation angle Θ, which using (6a) can be obtained from

$$\tan\Theta = \frac{2\Delta a}{r_0\varphi} = \frac{4}{\pi}\frac{R^2}{r_0^2} \qquad (7)$$

In separating a velocity change from u_0 to u_1 occurs in the cross-sections

$$F_0 = \pi a^2 \text{ and } F_1 = \pi a^2 - 2a\,\Delta a;\ u_0 F_0 = u_1(F_0 - 2a\,\Delta a)$$

or

$$u_1 = u_0\left(1 + \frac{2\Delta a}{\pi a}\right) = u_0\left(1 + \frac{4R^2}{\pi^2 a r_0}\right) \qquad (8)$$

However, according to the theory of flow separation, this change entails a pressure increase[1];

$$p_1 - p_0 = \frac{\gamma}{2g}(u_1^2 - u_0^2)\left(1 - \frac{4}{3}\tan\Theta\right) \qquad (9)$$

The effective portion of this involving $\tan\Theta$ gives

$$\Delta p' = \frac{2\gamma}{3g}\tan\Theta\,(u_1^2 - u_0^2) \qquad (10)$$

which is the pressure loss resulting purely from separation of flow. However, using (8), we may write

$$u_1^2 - u_0^2 = u_0^2\frac{4\Delta a}{\pi a} = u_0^2\frac{8R^2}{\pi^2 a r_0}$$

Thus, considering (7), (10) becomes

$$\Delta p' = \frac{u_0^2\gamma}{g}\frac{2}{3}\frac{4\Delta a}{\pi a}\tan\Theta = \frac{16}{3}\frac{u_0^2\gamma}{g}\frac{R^2}{\pi\varphi r_0 a} \qquad (10a)$$

which may therefore be neglected as being of second order in comparision with the pressure drop (4).

IV. Finally in connection with the remaining vortex loss (4), the pressure drop which must overcome the turbulent wall friction is

$$\Delta p'' = \frac{\pi r_0}{a}\lambda\frac{u_0^2\gamma}{2g} \qquad (11)$$

where λ is the roughness coefficient. Let us add this to (4), so that the total pressure loss in the curved tube is

$$\Delta p = \frac{\varphi}{\pi}\frac{u_0^2}{2g}\gamma\left(2\frac{a}{r_0} + \pi\lambda\frac{r_0}{a}\right) \qquad (12)$$

with a minimum value

$$\Delta p_0 = \frac{\varphi}{\pi}\frac{u_0^2\gamma}{2g}\sqrt{8\pi\lambda} \text{ when } \frac{a}{r_0} = \sqrt{\frac{\pi\lambda}{2}} \qquad (12a)$$

If ~~markthetheroughness~~ we set λ = 0.01 as the roughness of corresponding cast iron pipes, then

$$\frac{a}{r_0} = \sqrt{\frac{1.57}{100}} = 0.125 = \frac{1}{8}$$

for a minium value of Δp, which is in complete agreement with the results reached by Brightmore and Davis[2]

Abreviating, we may write;

[illegible] (13)

and in place of (12)

[illegible] (12a)

This provides the following table for [illegible] when [illegible] = 0.01.

r/a	1	2	3	4	6	8	10
	2.03	1.063	0.761	0.625	0.521	0.500	0.514

For the most useful quarter-circle bends or elbows, then with [illegible]

$\frac{\phi \zeta}{\pi}$ = 1.01 0.531 0.380 0.313 0.260 0.250 0.257

Therefore these numerical values are entirely within the range of observable values. They show a very strong rise with an increase of a/r and thus confirm the great influence of the double vortex in the cross-section.

1. Physik. Zeitschr. 30 ,77,1929.

2. Brightmore, Loss of pressure in water flowing through straight and curved pipes. Proc. Inst. Civ. Engineers, 169 ,315, 1907

Davis in Bemerkungen zu Schoder, Curve resistancein water pipes. Trans. Americ. Soc. Civ. Eng. 69, 63u. 109,1909.